

# Numerical Methods in FEKO

## Introduction

FEKO offers a wide spectrum of numerical methods and hybridizations, each suitable to a specific range of applications. Hybridization of numerical methods allows large and complex electromagnetic problems to be solved, thereby allowing solutions to problems that would be intractable with any individual method.

These state-of-the-art computational methods provide users with the ability to solve a broad range of electromagnetic (EM) problems. The EM simulation map shown below graphically presents the different solution methods and their range of applications.

- Method of moments (MoM) - Ideal for radiation and coupling analysis
- Multi-level fast multipole method (MLFMM) - Ideal for electrically large, full wave analysis
- Finite element method (FEM) - Ideal for problems with several dielectrics and waveguides
- Finite difference time domain (FDTD) - Well suited to modelling inhomogeneous materials and simulations over a wide frequency range
- Physical optics (PO) and large element physical optics (LE-PO) - Ideal for electrically very large radiation and scattering analysis
- Ray Launching geometrical optics (RL-GO) - Ideal for dielectric or metal, electrically very large scattering analysis
- Uniform theory of diffraction (UTD) - Ideal for electrically extremely large, perfect electrically conducting (PEC) structures
- Dedicated solver for wave propagation and radio network coverage analysis (WinProp)

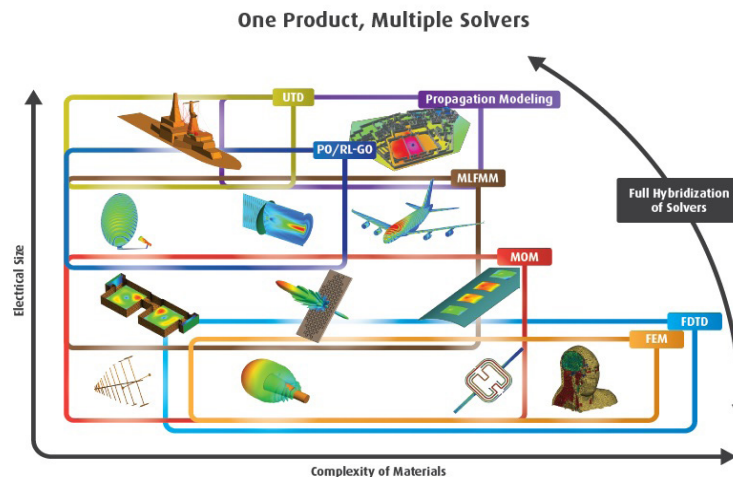


Figure 1: The EM simulation map gives a graphical illustration of the applicability of the various methods

## Method of Moments (MoM)

The MoM is a full wave solution of Maxwell's integral equations in the frequency domain. An advantage of the MoM is that it is a "source method", meaning that only the structure in question is discretized and not the free space surrounding it, as with "field methods".

Boundary conditions do not have to be set and memory requirements scale proportional to the size of the geometry and the required solution frequency.

Although the MoM is traditionally associated with open radiating problems involving PEC structures, lossy metals and dielectrics can be modelled in various ways. The surface equivalence principle (SEP) is used by default to model dielectrics in the MoM. For low frequency or extremely high permittivity materials, the volume equivalence principle (VEP) can be used. Dielectric materials can also be modeled as coatings on metal structures, infinite planar multilayer substrates, half spaces or thin dielectric sheets.

For the MoM, basis functions model the interaction between all triangles. The MoM treats each of the  $N$  basis functions in isolation, resulting in an  $N^2$  scaling of memory requirements (to store the impedance matrix) and  $N^3$  in CPU time (to solve the linear set of equations). As a result, processing requirements for MoM solutions scale rapidly with increasing problem size.

## Multilevel Fast Multipole Method (MLFMM)

The MLFMM is an alternative formulation of the MoM that enables the full wave current-based solutions of electrically large structures. Large models previously solved with the MoM can be solved by the MLFMM with no change required to the model.

Similar to the MoM, the MLFMM uses basis functions to model the interaction between all triangles. The MLFMM differs from the MoM in that it groups basis functions and computes the interaction between groups of basis functions, rather than between individual basis functions. The MLFMM's more efficient treatment of the same problem results in an  $N \cdot \log(N)$  scaling in memory and  $N \cdot \log(N) \cdot \log(N)$  in CPU time. In real applications this reduction in solution requirements between the MoM and MLFMM can range to orders of magnitude.

Significant effort has been invested in improving the parallel MLFMM formulation to achieve exceptional efficiency when distributing a simulation over multiple processors.

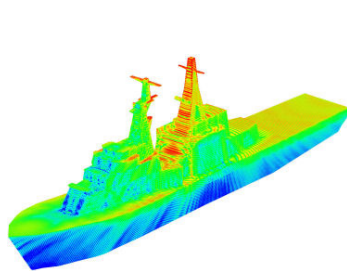


Figure 2: MLFMM analysis of a ship

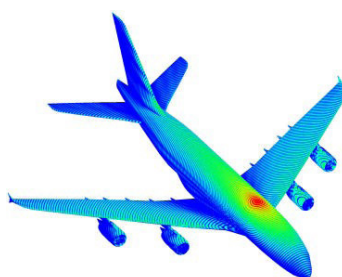


Figure 3: Antenna placement on a commercial aircraft

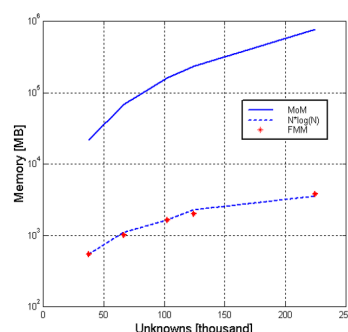


Figure 4: Scaling of the MLFMM memory compared to the MoM

## Finite Element Method (FEM)

The FEM is applicable to the efficient modeling of inhomogeneous dielectric bodies. It is also well suited to non-radiating microwave components such as shielded filters. The FEM is a volume discretization technique that uses tetrahedral mesh elements to represent arbitrarily shaped volumes accurately. Dielectric properties may vary between neighbouring tetrahedral mesh elements.

The FEM in FEKO usually invokes the hybrid FEM/MoM and not a pure FEM analysis. The FEM/MoM hybridisation features full coupling between metallic wires and surfaces in the MoM region and heterogeneous dielectric bodies in the FEM region. The MoM part of the solution is calculated first, which results in equivalent magnetic and electric currents that form the radiation boundary of the FEM region. This hybrid method incorporates the strengths of both the MoM and the FEM.

When a structure is bounded only by PEC surfaces and FEM modal ports, FEKO will recognize that the problem can be solved by just the FEM (fully sparse matrix solution), resulting in a reduction in memory and runtime.

For electrically large problems, the hybridized FEM/MLFMM can be used where the MLFMM solves the MoM part of the FEM/MoM problem efficiently.

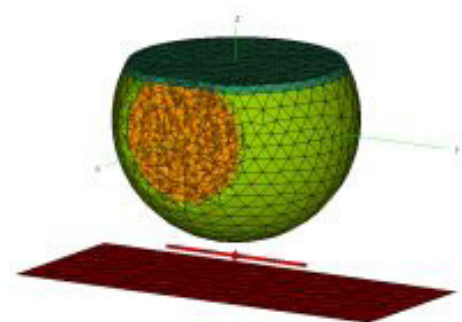


Figure 5: Full coupling between MoM and FEM regions

## Finite Difference Time Domain (FDTD)

The FDTD solution technique has gained popularity in computational electromagnetics (CEM) over the past decade due to its relatively straightforward and efficient formulation. As the name indicates, the FDTD is a time domain technique and Fourier transforms are applied to convert the native time domain results to the frequency domain.

The FDTD is best suited to problems that include highly inhomogeneous materials and therefore a popular choice in biomedical applications for the modeling of human phantoms. It is also a highly efficient solution for wideband problems, and is well suited to analyze broadband antennas. A single FDTD simulation with a pulsed excitation can be used to characterize a wideband frequency response of an antenna.

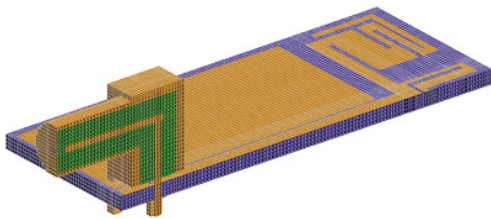


Figure 6: FDTD voxel mesh of a GSM antenna

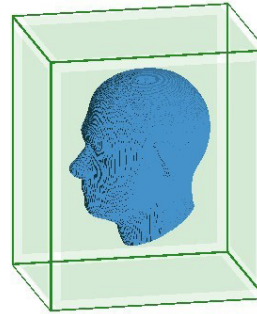


Figure 7: FDTD voxel mesh of a human head, with FDTD boundaries

## Physical Optics (PO)

PO is formulated for use in instances where electrically very large metallic or dielectric structures are modeled. It is an asymptotic high frequency numerical method of the same nature as the UTD, but based on currents and not rays. FEKO hybridizes the current-based accurate MoM with PO including bidirectional coupling between the MoM and PO regions. It triangulates a PO region, as it would for a MoM solution, making it a simple task to switch between solution options. In cases where the MoM part of the problem is electrically large, the PO hybridised with the MLFMM provides an efficient solution.

A practical example for PO would be to calculate the effect on the input impedance of a horn antenna (treated with the MoM), when placed in close proximity to a large structure (treated with the PO).

The large element physical optics (LE-PO) solution method is similar to the PO method, but allows larger elements to be used. Phase variation is taken into account across the elements. When the underlying model can be accurately described by larger elements, LE-PO is a suitable replacement for PO.

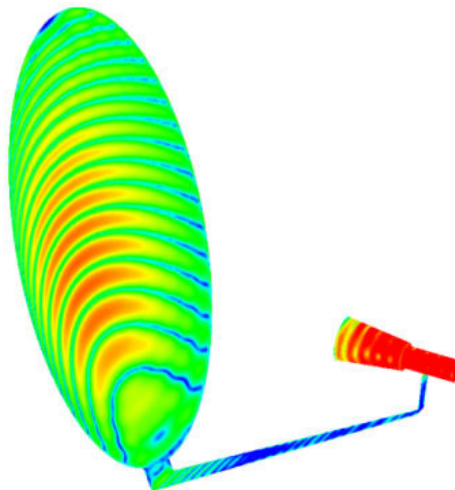


Figure 8: PO modeling of a reflector antenna with MoM modeling of the feed

## Ray Launching Geometrical Optics (RL-GO)

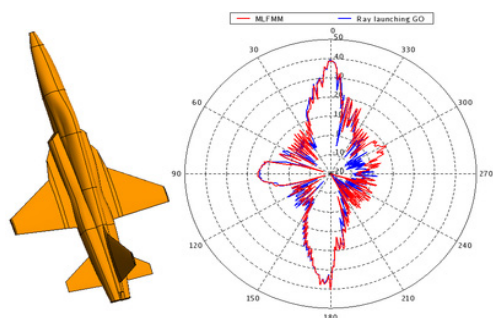


Figure 9: RCS of an aircraft at 1 GHz in the elevation plane: Comparison between MLFMM and RL-GO. RL-GO required 33 times less memory

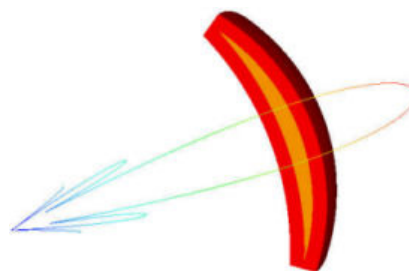


Figure 10: Ray launching through a lens

Ray launching geometrical optics is formulated for use in instances where electrically very large ( $> 20 \lambda$ ) metallic or dielectric structures are modeled. RL-GO is inherently well suited to the solution of large structure scattering problems such as radar cross section (RCS) analysis, since the “shooting and bouncing rays” approach is highly efficient for an arbitrary number of multiple reflections.

FEKO integrates the RL-GO method with the current-based MoM, by launching rays from each radiating MoM element. The ray interactions with metallic and dielectric structures are then modeled using Huygens sources placed on each ray contact point (for reflected, refracted and transmitted rays) on the material boundaries. The runtime and memory requirements scale almost perfectly for parallel processing, resulting in multi-core CPUs or cluster computers operating highly efficiently while solving RL-GO problems.

A typical application of the MoM/RL-GO hybrid method is the analysis of dielectric lenses. The source structure (for example a metallic antenna under a lens), may be modeled with the MoM and the large dielectric lens may be modeled with the RL-GO.

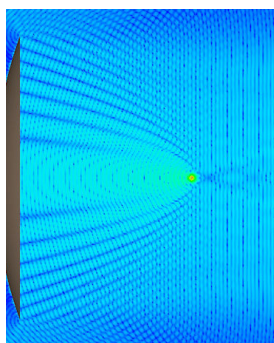


Figure 11: Reflector near field calculated with RL-GO

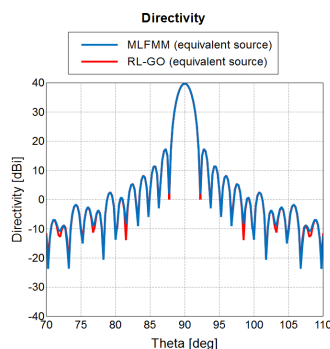


Figure 12: Comparison between RL-GO and MLFMM

Another application is the analysis of reflector antennas. The feed source can often be modeled in isolation to establish its radiation pattern. The pattern is then imported as an excitation for the RL-GO-based analysis of the reflector.

For accurate solutions to scattering problems, the premier method is full wave analysis with the MoM (or MLFMM). For electrically very large structures, an asymptotic method is required, as full wave methods result in prohibitive resource requirements. RL-GO is inherently well suited to the solution of large structure scattering problems, since the “shooting and bouncing rays” approach is highly efficient for an arbitrary number of multiple reflections.

## Uniform Theory of Diffraction (UTD)

The UTD is formulated for modeling electrically extremely large structures. The UTD is an asymptotic high frequency numerical method similar to the PO. Users will typically attempt a solution with the MoM, and when they realise that the structure is electrically too large to solve with their available resources (platform memory and time) they will turn to the MLFMM, PO and lastly UTD.

FEKO hybridizes the current-based accurate MoM with the UTD. Bidirectional coupling between the MoM and UTD is maintained in the solution (through modification of the interaction matrix) to ensure accuracy. Frequency has no effect on the memory resources required for solving a structure with UTD, given that only points of reflection from surfaces and diffraction from edges or corners are considered without meshing the structure.

Multiple reflections, edge and corner diffraction, double diffraction and creeping waves (along cylinders) are taken into account. Insight into the propagation of rays are provided in POSTFEKO during post-processing. Currently the numerical formulation of the UTD only allows it to be applied to flat polygonal plates with minimum edge length in the order of a wavelength, or to single cylinders. The UTD is well suited to the analysis of ships at radar frequencies, but less appropriate for analyzing complex objects with curved surfaces (such as automobiles).

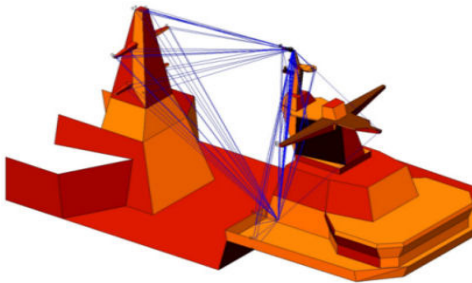


Figure 13: UTD modelling of cross-coupling on the superstructure of a modern naval vessel

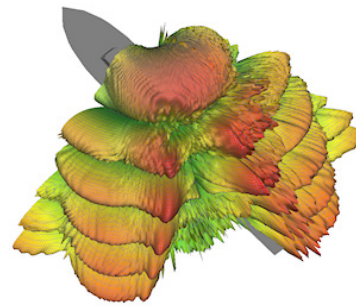


Figure 14: Analysis of the transmission patterns of an X-band radar mounted on a ship

## Additional Features and Extensions

FEKO offers optimised EM analysis and numerous features to increase productivity. Notable items include:

- **Higher order and curvilinear element support** that allows geometry to be meshed with larger triangles, which reduces the number of unknowns to be solved and the required memory.
- **Periodic boundary conditions** for analysing repetitive linear and planar structures, for example frequency selective surfaces (FSS).
- **Fast array analysis** for solving large, finite metallic antenna arrays.
- **Characteristic mode analysis** that provides physical insight into the radiating behaviour of objects, allowing for a systematic approach to antenna design and placement.
- **Error estimation and adaptive meshing** for assessing the quality of a mesh and adaptively refining the mesh in insufficiently meshed regions.
- **Model decomposition** through the substitution of complex sources and receivers with numerically efficient equivalent sources.
- **Numerical Green's function** for problems containing static and dynamic parts, allowing re-use of the static part of the solution in subsequent simulations.
- **Windscreen antenna solution method** that reduces the computational requirements by meshing only metallic elements, while fully taking into account the dielectric layers of glass in finite-sized windscreens.
- **Cable modeling** for including complex cable bundle networks in full wave simulations.
- **Adaptive cross approximation (ACA)** acceleration of the method of moments solution of complex problems, also effective for low frequency problems.
- **Parallel and graphics processing unit (GPU) support** for speeding up simulations.